(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9585 Roll No. | | | | | | | | | | |

B. Tech.

(Semester-I) Theory Examination, 2012-13

ELEMENTARY MATHEMATICS–I

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as per instructions. The symbols have their usual meaning.

Section-A

Attempt all parts of this question. Each part carries 2 marks. 2×10=20

1. (a) Evaluate :

$$\lim_{x \to 0} \frac{\sin 5x}{\sin 2x}.$$

(b) Find the differential coefficient of $\sin x . \cos x$.

(c) Find the differential coefficient of $(\sin x)^{\log x}$.

(d) Write the statement of the mean value theorem.

(e) Show $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

(f) Evaluate $\int \log x\,dx$.

(g) Find the order and degree of the differential equation :

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0.$$

(h) Write the general solution of differential equation :

$$\frac{dy}{dx} + Py = Q,$$

where P and Q are constants of functions of x only.

(i) Two coins are tossed once, find a sample space.

(j) A coin is tossed three times, consider the following events :

A : 'No head appears', B : 'Exactly one head appears', and C : 'At least two heads appear'.

Do they form a set of mutually exclusive and exhaustive events?

Section-B

Attempt any *three* parts of this question. Each part carries 10 marks. 10×3=30

2. (a) Show that :

$$\lim_{x\to 0} (1+x)^{1/x} = e.$$

(b) Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

(c) Evaluate the definite integral $\int_a^b \sin^2 x\,dx$ as the limit of a sum.

(d) Solve :

$$\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}.$$

(e) If A and B are any two events of a sample space S and F is an event of S such that $P(F) \neq 0$, then show :

$$P((AB)|F) = P(A \cup F) + P(B|F) - P((AB) \cap F).$$

Section-C

All questions of this Section are compulsory. Attempt any *two* parts from each question. Each question carries 10 marks. 10×5=50

3. (a) Find the value of :

$$\lim_{x\to 2}\left[\frac{x^3-4x^2+4x}{x^2-4}\right].$$

(b) Find the derivative of f from the first principle, where $f(x)=x+\frac{1}{x}$.

(c) Compute $\frac{dy}{dx}$, where :

$$y=\sin x+\tan(x^2)+x^4.$$

4. (a) If $x=a\cos^3\theta$, $y=a\sin^3\theta$, find $\frac{dy}{dx}$.

(b) Determine the values of a and b for which the function :

$$f(x)=\begin{cases} ax^2+b\ , & x\le 0 \\ \dfrac{-3}{x^2+1}+1, & x>0 \end{cases}$$

is continuous at $x=0$.

(c) Prove that :

$$b^n-a^n<nb^{n-1}(b-a),\ b>a\ .$$

5. (a) Evaluate :

$$\int_0^{\pi}\log(1+\cos x)\,dx\ .$$

(b) Find the area of the region bounded by the parabolas $y^2=4ax$ and $x^2=4ay$.

(c) Evaluate :

$$\int\frac{x^5}{\sqrt{1+x^3+x^6}}\,dx\ .$$

6. (a) Solve :

$$\frac{dy}{dx}+1=e^{x+y}\ .$$

(b) Solve $yp^2+(x-y)\,p-x$, where $p=\frac{dy}{dx}$.

(c) Solve :

$$\frac{dy}{dx}=xy+x+y+1.$$

7. (a) If A and B are two independent events, then show that the probability of occurrence of at least once of A and B is given by :

$$1-P(A').P(B').$$

(b) Find the variance of the number obtained on a throw of an unbiased die.

(c) Bag I contain 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is random drawn from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.